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# SPECIFICATION

# ADAPTIVE FILTER TO REDUCE MULTIPATH

## Cross Reference to Related Applications

This method of reducing multipath avoids the cross term noise produced by other methods [Refs. 1 through 10]. These other methods correlate (cross correlate or auto correlate) two versions of the signal each containing the direct path and multipath reflections. A correlation peak between the direct path and a reflection gives a measurement of the relative delay between the direct path and the reflected path and the amplitude and phase of the reflection. As the relative delay between the two signals is moved from this peak, other correlation peaks are produced when there is correlation between the reflections, i.e., when the relative delay equals the difference of the delays for two objects. This produces noise which interferes with the measurement since there are many possible combinations. When one delays one of the signals and subtracts it from the other with the correct amplitude and phase, he tends to cancel the multipath. The other reflections, however, are dragged along with it and produce noise. These noises are avoided by the invention described in this disclosure.

## Statement Regarding Federally Sponsored R&D



No Federal funds were used to conceive or develop this invention.

## Reference to Appendix Containing Computer Program

The Appendix contains listings of computer programs written in the APL language and a sample computer run. The programs embody the basic features of this invention. The computer run is a demonstration of its performance on simulated data.

The program NORM generates simulated data. The program DELRES calculates the delay residuals. The program NLS minimizes the residuals defined in DELRES by nonlinear least squares.

The program NLS is first edited so as to output the residuals as RR. The program NORM is used to generate 100000 mean zero variance 1 random numbers in X5. This represents a 50000 Hz broadband signal sampled at 100000 samples per second with r.m.s. (root mean square) of 1. Two multipaths are simulated both of 0.1 magnitude. One is delayed 1 sample; the other is delayed 3 samples. The result is put into XX5. The signal to multipath noise ratio is calculated to be 17 dB. The NLS program is applied to XX5 and the two multipath noises are canceled to a level of 51 dB signal to multipath noise ratio.

## Background

The direct path of a radio signal from transmitter to receiver is frequently interfered with by reflections of the signal from stationary and moving objects. This is called multipath noise. This invention utilizes a new adaptive filter technique to reduce multipath noise. This method of reducing multipath avoids the cross term noise produced by other methods [Refs. 1 through 10]. These other methods correlate (cross correlate or auto correlate) two versions of the signal each containing the direct path and multipath reflections. A correlation peak between the direct path and a reflection gives a measurement of the relative delay between the direct path and the reflected path and the amplitude and phase of the reflection. As the relative delay between the two signals is moved from this peak, other correlation peaks are produced when there is correlation between the reflections. This produces noise which interferes with the measurement. When one delays one of the signals and subtracts it from the other with the correct amplitude and phase, he tends to cancel the multipath. The other reflections, however, are dragged along with it and produce noise. These noises are avoided by the invention described in this disclosure.

## Brief Summary

A nonlinear least squares method measures the delay, Doppler shift and amplitude of the multipath due to each object and subtracts a very accurate reconstruction of each multipath signal from the noisy signal. If an object is a target, its range, range rate and magnitude is got from the measured multipath delay, Doppler shift and amplitude. Position and velocity of the target can be obtained by geometric triangulation with multiple transmitters. Target angle can be measured by the relative phase of the corresponding filter coefficients across multiple antennas. The system can be used on a surveillance aircraft to cancel ground reflections and measure targets.

## Brief Description of Drawing

The Figure is a block diagram illustrating the adaptive filter of the invention to reduce multipath in a radio transmitted signal wherein the multipath is caused by reflections from fixed and moving objects, interferers, or targets.

The method works best on white signals; therefore, prewhitening and post unwhitening filters are employed.

The differential delay and Doppler shift for each target is measured by the filter weight $x_{mn}$ which corresponds to the differential delay $nD$ and Doppler shift $mf$ caused by that target. The magnitude of the filter weight is a measure of the strength of the target.

The quantity labeled "Residual" in the Figure is the received signal minus the result of application of the filter weights $w_1, w_2, w_3, x_{11}, x_{12}, x_{13} \cdots, x_{21}, x_{22}, x_{23} \cdots x_{31}, x_{32}, x_{33} \cdots$ . A nonlinear least squares method is used to pick the filter weights which minimize the mean squares of the "Residual". The result is a clean version of the original signal which is the received signal minus the multipath reflections.

The filter weight $x_{nm}$ corresponding to a moving target is a measure of the differential range $nDc$ and range rate $mfc$ where $c$ is the speed of light. The system, with a common antenna, can be duplicated to receive two signals each from each of two widely separated transmitters. Geometric triangulation can be used to measure the two dimensional position and velocity of the target. Triplication can be used for three dimensional position and velocity.

Another use of the system is to detect the angle of moving targets. If multiple antennas are provided, each connected with a system like that shown in the Figure, the angle of a moving target causing a multipath reflection to the receiving antennas can be measured by using the relative phase of the corresponding delay and Doppler complex coefficients across the several antennas .

The receiving antennas, receivers and processing system can be placed in a surveillance aircraft the position and velocity of which is obtained by an accurate navigation system such as GPS. Objects on the ground, interferers and targets causing multipath reflections can be processed by the system. Target position and velocity can be obtained by adding the position and velocity of the surveillance aircraft to the measured position and velocity of the target.

## Detailed Description

This method of reducing multipath avoids the cross term noise produced by other methods [Refs. 1 through 10]. These other methods correlate (cross correlate or auto correlate) two versions of the signal each containing the direct path and multipath reflections. A correlation peak between the direct path and a reflection gives a measurement of the relative delay between the direct path and the reflected path and the amplitude and phase of the reflection. As the relative delay between the two signals is moved from this peak, other correlation peaks are produced when there is correlation between the reflections. This produces noise which interferes with the measurement. When one delays one of the signals and subtracts it from the other with the correct amplitude and phase, he tends to cancel the multipath. The other reflections, however, are dragged along with it and produce noise. These correlation noises are avoided by the invention described in this disclosure.

The Figure is a block diagram illustrating the adaptive filter of the invention to reduce multipath in a radio transmitted signal wherein the multipath is caused by reflections from fixed and moving objects, interferers, or targets.

The method works best on white signals; therefore, prewhitening and post unwhitening filters are employed.

The differential delay and Doppler shift for each target is measured by the filter weight $x_{nm}$ which corresponds to the differential delay $nD$ and Doppler shift $mf$ caused by that target. The magnitude of the filter weight is a measure of the strength of the target.

The quantity labeled "Residual" in the Figure is the received signal minus the result of application of the filter weights $w_1, w_2, w_3, x_{11}, x_{12}, x_{13} \cdots, x_{21}, x_{22}, x_{23} \cdots x_{31}, x_{32}, x_{33} \cdots$ . A nonlinear least squares method is used to pick the filter weights which minimize the mean squares of the "Residual". The result is a clean version of the original signal which is the received signal minus the multipath reflections.

Refer to the Figure. The signal is modulated and amplified by the transmitter, Tx [Ref. 17, ROHDE, p.14]. After transmission from the antenna most of the power takes the direct path to the receive antenna. Some power, however, is reflected from stationary and moving objects such as buildings and aircraft. This tends to add or subtract from the direct path causing what is known as multipath interference noise[Ref. 18, SKOLNIK, p. 18.46]. This invention removes the multipath noise from the received signal.

The signal at the receive antenna is amplified and demodulated by the receiver, Rx [Ref. 17, ROHDE, p. 361 ][Ref. 19, VAN VALKENBURG, p. 23-6]. It is then passed through a presampling filter, sampled and converted from analog to digital [Ref. 19, VAN VALKENBURG, p. 28-7]. These real digital samples have a Fourier transform the real part of which is symmetric about zero frequency and the imaginary part of which is antisymmetric about zero frequency. The real samples are first frequency shifted by multiplying by $e^{-j2\pi Ft}$ where $t$ is time, $F = \frac{1}{2D}$ and $2/D$ is the sampling frequency [Ref. 14, PAPOULIS, p. 15]. These complex samples are then passed through a low pass digital filter with a maximum frequency of F [Ref. 16, RABINER, p. 100][Ref. 13, OPPENHEIM, p. 261]. The output of the low pass filter is resampled at a frequency of $1/D$ , i.e., every other sample, to obtain the complex samples of the received signal.

The received signal, after demodulation, analog to digital conversion and conversion to complex samples, can be modeled as

$$q_i = s_i + \sum_{n=1}^{N} \sum_{m=-M}^{M} a_{mn} e^{-j2\pi mfnD} s_{i-n} E^m + \eta_i$$

where

$s_i$ = original signal

$i$ = sample index

$D$ = sampling time interval

$n$ = time delay index

$f$ = Doppler shift frequency interval

$m$ = Doppler shift index

$a_{mn}$ = Multipath amplitude coefficient

$j$ = unit imaginary number

$e$ = base of natural logarithms

$\pi$ = pi

$E = e^{j2\pi ft}$

$t = iD$

$\eta_i$ = white noise

From the Figure, the residual is

$$r_i = q_i - \sum_{l=1}^{L} w_l q_{i-l} - \sum_{n=1}^{N} \sum_{m=-M}^{M} x_{mn} r_{i-n} E^m$$

where the $w_l$ are whitening filter coefficients and the $x_{mn}$ are the delay-Doppler filter coefficients. This equation corresponds to that of Box [Ref. 11, BOX, p. 498]. One can minimize the root mean squared (r.m.s.) residual

$$\sqrt{\frac{1}{I}\sum_{i=1}^{I} r_i^2}$$

over

$$w_1, w_2, \cdots, w_L, x_{-M1}, x_{-M2}, \cdots, x_{-MN}, x_{-M+11}, x_{-M+12}, \cdots, x_{-M+1N}, \cdots, x_{MN}$$

by nonliear least squares [Ref. 11, BOX, p. 504, p. 505] [Ref. 12, MARQUARDT, p. 431].

To show that this removes the Multipath and restores the original signal, define the whitened signal as

$$p_i = s_i - \sum_{l=1}^{L} w_l s_{i-l}$$

and the modified noise as

$$\varsigma_i = \eta_i - \sum_{l=1}^{L} w_l \eta_{i-l}$$

Then, the residual is

$$r_i = p_i + \sum_{n=1}^{N} \sum_{m=-M}^{M} a_{nm} e^{-j2\pi mfnD} p_{i-n} E^m + \varsigma_i - \sum_{n=1}^{N} \sum_{m=-M}^{M} x_{nm} r_{i-n} E^m$$

The root mean square residual is minimized when

$$x_{nm} \approx a_{nm} e^{-j2\pi mfnD}$$

and

$$r_i \approx p_i + \varsigma_i$$

Note that only delayed samples of the whitened signal $p_i$ , i.e., $p_{i-n}$, $n \neq 0$, are subtracted from $p_i$ , so that it is not possible to cancel the whitened signal $p_i$ since it is uncorrelated with delayed versions of itself.

Adding the above two equations which describe the action of the whitening filter, one obtains

$$r_i \approx (s_i + \eta_i) - \sum_{l=1}^{L} w_l (s_{i-l} + \eta_{i-l})$$

Solving this equation, one gets

$$s_i + \eta_i \approx r_i + \sum_{l=1}^{L} w_l (s_{i-l} + \eta_{i-l})$$

which, from the Figure, gives the output of the unwhitening filter. This is the desired result which restores the original signal without multipath. The $x_{nm}$ imply target differential range, range rate and magnitude.

A simple test of this concept was made with the computer program shown in the Appendix. The Marquardt method for nonlinear least squares [Ref. 11, BOX, p. 504, p. 505] [Ref. 12, MARQUARDT, p. 431] was programmed in the APL language in program NLS. The equation for the residual was programmed in DELRES with

$x_{mn} = 0$ when $m \neq 0$

and

$w_l \equiv 0$

If

$w_l \neq 0$

this corresponds to the equation of Box [Ref. 11, BOX, p. 498] and is a general recursive digital filter [Ref. 15, PAPOULIS, p. 45].

The original signal $s_i$ was generated with a Gaussian random number generator in program NORM with root mean square (r.m.s.) of 1 and $I = 100000$. This corresponds to one second of a 50000 Hz bandwidth signal represented by 100000 real samples. The data is white, zero mean, Gaussian with r.m.s. = 1.

The received signal $q_i$ was generated with

$a_{01} = a_{03} = 0.1$ , otherwise = 0

and

$\eta_i = 0$

This corresponds to two multipaths both of magnitude 0.1, one is delayed 1 sample, the other is delayed 3 samples. The white signal itself provides an effective noise which perturbs the estimates.

The process converged in three iterations, the final r.m.s. residual = 1.002370568. The three coefficients $x_{01}, x_{02}, x_{03}$ are shown to be approximately = 0.1, 0.0, 0.1 matching $a_{01}, a_{12}, a_{03}$ with r.m.s. errors of about 0.003. The mean residual was about 0.0087. The original signal to multipath noise ratio was 17 dB. After processing, the two multipaths were canceled to a signal to multipath noise ratio of 51 dB.

The filter weight $x_{mn}$ corresponding to a moving target is a measure of the differential range $nDc$ and range rate $mfc$ where $c$ is the speed of light. The system, with a common antenna, can be duplicated to receive two signals each from each of two widely separated transmitters. Geometric triangulation [Ref. 18, SKOLNIK, p.25.5, p.25.13] can be used to measure the two dimensional position and velocity of the target. Triplication can be used for three dimensional position and velocity.

Another use of the system is to detect the angle of moving targets. If multiple antennas are provided, each connected with a system like that shown in the Figure, the angle of a moving target causing a multipath reflection to the receiving antennas can be measured by using the relative phases of the corresponding delay and Doppler complex coefficients across the several antennas [Ref. 18, SKOLNIK, p.3.34, p.3.35].

The receiving antennas, receivers and processing system can be placed in a surveillance aircraft the position and velocity of which is obtained by an accurate navigation system such as GPS. Objects on the ground, interferers and targets causing multipath reflections can be processed by the system. Target position and velocity can be obtained by adding the position and velocity of the surveillance aircraft to the measured position and velocity of the target.

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# APPENDIX

## Computer Programs and Sample Run

```
∇NORM[⎕]∇
[0]    X←NORM N;Z;T;R
[1]    ⍝ ;;
[2]    ⍝ 961204.1712
[3]    ⍝ GENERATE N NORMAL RANDOM VARIABLES, MEAN 0, SIGMA 1
[4]    ⍝ ⎕RL IS SEED
[5]     Z←2,⌈N÷2
[6]     Z←Zρ(÷10000)×?(×/Z)ρ10000
[7]     T←(○2)×Z[1;]
[8]     R←(-2×⍟Z[2;])*0.5
[9]     X←N↑,(1 1∘.×R)×2 1∘.○T
    ∇ 1996-12-10 11.00.01 (GMT-4)

       ∇DELRES[⎕]∇
[0]    R←Y DELRES X;N;I
[1]    ⍝ ;;
[2]    ⍝ 000323.2241
[3]    ⍝ DELAY RESIDUALS
[4]     N←ρX
[5]     R←0×Y
[6]     R[⍳N]←Y[⍳N]
[7]     I←0
[8]    NEXTI:I←I+1
[9]     R[N+I]←Y[N+I]-R[N+I-⍳N]+.×X
[10]    →NEXTI×⍳I<(ρY)-N
[11]    R←N↓R
    ∇ 2000-03-25 22.51.47 (GMT-4)

       ∇NLS[⎕]∇
[0]    XE←XD NLS RES;X;ΔX;R;P;J;ΔR;A;B;D;EM;E;I;EE;AA
[1]    ⍝ ;RES;
[2]    ⍝ 800225.0914 800820 000321.1532
[3]    ⍝ NONLINEAR LEAST SQUARES
[4]    ⍝ CHAR. VECT. RES IS RESIDUAL FUNCTION NAME
[5]     X←XD[1;]
[6]     ΔX←XD[2;]
[7]     EE←''
[8]     I←0
[9]    NEXTI:I←I+1
[10]    R←⍎RES,' X'
[11]   ⍝ (R A B)←⍎RES,'(X ΔX)'
[12]   ⍝ EE←EE,(÷/+/(R*2),[0.5]1)*0.5
[13]    (÷/+/(R*2),[0.5]1)*0.5
[14]   ⍝ ANALYTIC RESIDUAL PARTIALS
[15]   ⍝ P←⍎(RES,'P'),' X'
[16]   ⍝→ANPAR
[17]   ⍝ CALCULATE R, A AND B
[18]   ⍝ (R A B)←⍎RES,'(X ΔX)'
[19]   ⍝ →CALCRAB
[20]   ⍝ NUMERIC RESIDUAL PARTIALS
[21]    P←''
[22]    J←0
[23]   NEXTJ:J←J+1
[24]    ΔR←(⍎RES,' X+ΔX[J]×J=⍳ρX')-R
[25]    P←P,ΔR÷ΔX[J]
```

```
[26]   →NEXTJ×⍳J<⍴X
[27]   P←⍉((⍴X),(⍴P)÷⍴X)⍴P
[28]  ANPAR:A←(⍉P)+.×P
[29]   B←(⍉P)+.×R
[30]  CALCRAB:
[31]   D←÷(((0 1+⍴A)⍴A)[;1]+1E¯10)*0.5
[32]   AA←(A×D∘.×D)+1E¯10×(⍴A)⍴1,0×A
[33]   X←X-D×(D×B)⌹AA
[34]   →NEXTI×⍳I<3
[35]   R←⍎RES,' X'
[36]  ⍝ (R A B)←⍎RES,'(X ∆X)'
[37]   EM←÷/+/R,[0.5]1
[38]   E←(÷/+/(R*2),[0.5]1)*0.5
[39]   XE←(X,EM),[0.5]E×(((0 1+⍴AA)⍴(D∘.×D)×⌹AA)[;1],1)*0.5
[40]  ⍝ EE,E
[41]   E
[42]  ⍝ ' '
[43]  ⍝ XE
[44]  ⍝ →NEXTI×⍳I<20
[45]  ⍝ RR←R
    ∇ 2000-03-29 23.00.55 (GMT-4)

      ∇NLS[⎕45]
[45]  ⍝ RR←R
[45]   RR←R
[46]  ∇


      ⍴X5←NORM 100000
100000
      ⎕←S←(÷/+/((X5)*2),[.5]1)*.5
1.002357739
      ⍴XX5←X5[3+⍳99997]+.1×X5[(⍳99997)∘.+¯1+⍳4]+.×1 0 1 0
99997
      ⎕←N←(÷/+/((XX5-3↓X5)*2),[.5]1)*.5
0.1418755036
      20×10⍟S÷N
16.98230664
      (2 3⍴0 0 0 .001 .001 .001)NLS 'XX5 DELRES'
1.011856929
1.002412348
1.00237057
1.002370568
0.09896411424  0.001084301862 0.09780701046  0.008745036907
0.003147221067 0.003163105505 0.003146911359 1.002370568
      ⎕←N←(÷/+/((RR-6↓X5)*2),[.5]1)*.5
0.002841730873
      20×10⍟S÷N
50.94879605
```